Semantic Phasor Theory: A Unified Spectral Model of Meaning and Cognitive Structure

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ABSTRACT

This work introduces semantic phasor theory, a mathematical framework that represents meaning as spectral structure. The theory extends spectral semantics by unifying concepts from AI, cognitive science, and signal processing into a single operator-theoretic model. Within this framework, any semantic domain can be expanded in a spectral basis, with complex coefficients interpreted as semantic phasors whose magnitudes and phases encode the contribution and alignment of each semantic mode. This generalizes classical phasor analysis to abstract meaning spaces and provides a coordinate-free alternative to high-dimensional embedding methods. Semantic similarity, interference, and compositionality emerge naturally from the spectral decomposition. A worked example on a finite semantic subdomain demonstrates the full semantic-phasor pipeline. The result is a rigorous and unified foundation for modeling meaning through spectral operators.

Key-words: semantic phasor theory; spectral semantics, operator theory; Hilbert space models of meaning; semantic modes; spectral decomposition; complex-valued semantic fields; semantic similarity; semantic resolution; semantic holography; modulation and ambiguity functions.

1. Introduction

Representing meaning in a mathematically principled way remains one of the central open challenges in the study of language, cognition, and artificial intelligence. Contemporary approaches—ranging from distributional semantics to high-dimensional neural embeddings—offer powerful empirical tools yet lack a unified mathematical foundation capable of explaining semantic structure, compositionality, and relational meaning within a single coherent framework.[1,2,3] This paper develops such a foundation by introducing Semantic Phasor Theory, a unified operator-theoretic model in which meanings are represented as spectral structures generated by self-adjoint operators on Hilbert space.

The central idea is to treat a semantic domain Ω as the base space of a complex-valued field, and to encode semantic relations in a semantic operator T acting on the Hilbert space $\mathcal{H} = L^2(\Omega)$ or its finite-dimensional analogues. By the spectral theorem, a self-adjoint operator T on $\mathcal{H}$ admits an orthonormal basis of eigenfunctions $\{e_1, e_2, \dots\}$, where each e_k represents an individual semantic mode in the spectral basis. Any meaning $m \in \mathcal{H}$ can then be expressed as a superposition:

$$m = \sum_k c_k \, e_k,$$

where the complex coefficients

$$c_k = | \, c_k \, | \, e^{i\theta_k}$$

are interpreted as semantic phasors.[4] The magnitudes $| \, c_k \, |$ quantify the contribution of each semantic mode, while the phases θ_k encode alignment, interference, and relational structure, in analogy with phasor and holographic representations in signal processing and optics. This spectral-phasor representation generalizes classical phasor analysis from signal processing to abstract semantic domains, providing a mathematically rigorous and coordinate-free model of meaning.

This operator-theoretic formulation yields a unified theory that connects ideas from functional analysis, spectral geometry, signal processing, and semantic representation. Unlike conventional embedding methods, which rely on high-dimensional vector spaces with opaque learned coordinates, the semantic-phasor framework derives its representational structure from the intrinsic spectral properties of T. Semantic similarity, compositionality, and interference effects arise naturally from the geometry of the spectral decomposition, offering a principled alternative to purely empirical embedding techniques. As will be shown below, the associated semantic transfer function provides (i) a complete description of how meaning is transformed, (ii) a mechanism for analyzing semantic resolution and ambiguity, (iii) a basis for defining semantic signal-to-noise ratio (semantic SNR) and semantic bandwidth, and (iv) a foundation for semantic holography and semantic phasor transformers, by analogy with transfer-function analyses in optics, radar, and holography.[5] By grounding semantic inference in spectral operator theory, this framework unifies semantic cognition with the mathematics of optics, radar, and holography while extending beyond the limitations of Fourier-centric models.

Although semantic embeddings in contemporary language models typically inhabit high-dimensional continuous spaces, it is often analytically useful to examine a restricted semantic subdomain. Any finite subset of concepts—for example, A, B, C—can be viewed as a coarse-grained projection of a much larger embedding space, obtained by selecting a small group of tokens and considering only their pairwise relations. The resulting finite domain inherits its structure from the underlying embeddings through a similarity or adjacency operator yet remains simple enough to permit explicit spectral analysis. A worked example on such a finite subdomain illustrates the full semantic-phasor pipeline—from domain specification to operator construction, spectral decomposition, phasor representation, and reconstruction—demonstrating how the theory operates in practice.

The result is a mathematically rigorous and conceptually unified framework for representing meaning as spectral structure. By grounding semantics in operator theory, Semantic Phasor Theory provides a foundation capable of supporting both theoretical analysis and practical applications across language modeling, cognitive science, and signal-based representations of information.

2. Semantic Fields and the Semantic Hilbert Space

2.1 Semantic Fields

Let $d \in \mathbb{N}$ denote the intrinsic dimensionality of the semantic domain, and define a semantic field as a complex-valued function

$$B: \mathbb{R}^d \to \mathbb{C}.$$

For each point $x \in \mathbb{R}^d$, the complex value $B(x)$ encodes the semantic content associated with that point in the domain.

This representation allows semantic content to exhibit interference, coherence, and phase-dependent interactions, thereby providing a natural mathematical mechanism for modeling ambiguity, compositionality, and contextual modulation, in analogy with complex wave fields in physics and signal processing.[6]

2.2 Hilbert-Space Structure

Semantic fields serve as the foundation upon which semantic operators act. Their spectral decomposition reveals the semantic modes that govern how meaning evolves under inference, filtering, and reconstruction. The Hilbert-space formulation provides the mathematical foundation for spectral decompositions, operator-based semantic representations, and the introduction of semantic phasors as complex spectral coefficients.

Semantic fields are elements of the complex Hilbert space

$$\mathcal{H} = L^2(\mathbb{R}^d),$$

consisting of equivalence classes of complex-valued functions on $\mathbb{R}^d$ whose squared magnitude is integrable over the domain.

The inner product on $\mathcal{H}$ is defined as follows. For two semantic fields $B_1, B_2 \in \mathcal{H}$, the inner product $\langle B_1, B_2 \rangle$ is given by:

$$\langle B_1, B_2 \rangle = \int_{\mathbb{R}^d} B_1(x)\, \overline{B_2}(x)\, \mathrm{d}x,$$

where $\overline{B_2}(x)$ denotes the complex conjugate of $B_2(x)$. This complex inner product captures not only the magnitude of semantic overlap but also the relative phase between semantic components. This phase sensitivity enables the framework to represent relational meaning, contextual shifts, and semantic coherence in a mathematically principled way, paralleling complex-valued embedding methods in machine learning and knowledge-graph modeling.

This structure provides:

- a norm $\| B \| = \sqrt{\langle B, B \rangle}$ measuring total semantic "energy";
- a geometry in which semantic similarity is expressed through complex inner products;
- a linear space in which semantic superposition and interference are well defined.

The complex structure of $B(x)$ decomposes semantic information into two complementary components. The amplitude $| B(x) |$ quantifies semantic salience, strength, or intensity, while the phase $\arg(B(x))$ captures relational orientation, contextual alignment, or inferential stance, analogous to modulus-and-phase roles in complex embeddings and holographic.

2.3 The Semantic Fourier Transform

The Fourier transform:

$$\hat{B}(k) = \int_{\mathbb{R}^d} B(x)\; e^{-2\pi i k\cdot x}\, \mathrm{d}x$$

maps semantic fields from the spatial domain to a spectral domain indexed by frequencies $k \in \mathbb{R}^d$. The inverse transform is given by:[7]

$$B(x) = \int_{\mathbb{R}^d} \hat{B}(k)\, e^{2\pi i k\cdot x}\, \mathrm{d}k$$

which reconstructs the semantic field from its spectral representation. Semantic modes correspond to frequency components k, and semantic resolution is governed by the spectral support of $\hat{B}$, mirroring how bandwidth and resolution are linked in classical signal processing and imaging.[8]

Taken together, these constructions show that the semantic phasor—complex spectral coefficients with interpretable amplitude and phase—provides a mathematically rigorous bridge between physics-based signal-processing formalisms and the non-physical, phenomenological processing of meaning in human cognition.[9,10]

3. Semantic Operators

3.1 Bounded Linear Operators

3.1 Semantic operators and spectral inference

Semantic inference is modeled as the action of bounded linear operators on the semantic Hilbert space $\mathcal{H}$, where semantic fields $B(x)$ encode amplitude and phase structure. A semantic operator is a bounded linear map $T: \mathcal{H} \rightarrow \mathcal{H}$, and the central insight of the semantic-phasor framework is that the behavior of such operators is governed by their spectral decomposition. When T is diagonalizable (for example, normal on a finite-dimensional subspace), it admits an orthonormal basis of eigenvectors $\{v_k\}$ such that math.

$$Tv_k = \lambda_k v_k,$$

with complex eigenvalues $\lambda_k = r_k e^{i\theta_k}$.

These eigenpairs define the semantic phasor modes of the system: the eigenvectors $v_k$are semantic directions along which meaning evolves independently, the magnitudes $r_k =| \lambda_k |$ encode semantic gain (amplification or attenuation), and the phases $\theta_k = \arg(\lambda_k)$ encode semantic rotation or relational shift, in analogy with complex gain and phase in wave and communication systems. Any semantic field B can be decomposed as

$$B = \sum_k \hat{B}\,(k)\, v_k, \hat{B}(k) = \langle v_k, B \rangle,$$

and the action of Tbecomes

$$TB = \sum_k \lambda_k\ \hat{B}(k)\, v_k. [][]$$

This representation shows that semantic inference is fundamentally a spectral process: each semantic mode evolves independently under a complex scaling determined by λ_k. The operator amplifies some modes, suppresses others, and rotates semantic phase across the spectrum, mirroring the role of eigenmodes in optical propagation, radar filtering, and holographic reconstruction. The spectral form of T thus provides a principled mechanism for analyzing semantic resolution, semantic noise, semantic ambiguity, and semantic reconstruction, and forms the mathematical foundation for semantic transfer functions, semantic holography, and the semantic phasor transformer architecture developed in later sections.

3.2 Semantic transfer functions

A semantic transfer function is a complex-valued function $\hat{T}(k)$ that acts multiplicatively in spectral space. This is the semantic analogue of optical modulation transfer functions, radar matched-filter responses, and convolution kernels in signal processing, all of which are characterized by frequency-domain gains and phases. Semantic operators act on semantic fields by amplifying, suppressing, or rotating specific semantic modes; in the spectral-mode framework, these effects are captured by the semantic transfer function, which is defined directly from the operator's eigenvalues.[11]

Let

$$T = \sum_k \lambda_k \, P_k$$

be the spectral decomposition of a diagonalizable semantic operator, where $\lambda_k = r_k e^{i\theta_k}$ and $P_k$is the projector onto the semantic mode v_k. For any semantic field B, its spectral representation is $\hat{B}(k) = \langle v_k, B \rangle$, and in this basis the action of T becomes:

$$TB = \sum_k \lambda_k \ \hat{B}(k) \, v_k . []$$

We focus on linear operators that are diagonalizable in a spectral basis (e.g., Fourier or other semantic eigenbasis), so that their action is fully characterized by the semantic transfer function

$$\hat{T}(k) = \lambda_k .$$

Here the magnitude $r_k = | \lambda_k |$specifies semantic gain, the phase $\theta_k = \arg(\lambda_k)$ specifies semantic rotation, and the eigenvector $v_k$specifies the semantic direction being transformed, paralleling complex-valued embedding models where relations act as rotations in phase. This formulation generalizes the modulation transfer function in optics and the matched-filter response in radar, where operators act by independently scaling and rotating spectral components.[12]

3.2a Interpretation

The semantic transfer function $\hat{T}(k)$ provides a principled mechanism for semantic filtering (emphasizing or suppressing modes), semantic resolution (which distinctions remain detectable), semantic coherence (how phase relationships are preserved or distorted), and semantic inference (transforming meaning through structured gain and rotation). Modes with large $| \hat{T}(k) |$ dominate interpretation, while modes with small $| \hat{T}(k) |$are attenuated; the phase response

$\arg\left(\hat{T}(k)\right)$ determines how relational structure is shifted across modes, analogous to phase-based relational encodings in complex embeddings and HRR-style models.

3.2b Fourier-domain transfer functions as a special case

When the semantic operator T is convolutional, $TB = a * B$, the diagonalizing basis is the Fourier basis. In this case, the semantic transfer function reduces to the classical Fourier multiplier:

$$\hat{T}(k) = \hat{a}(k), \widehat{TB}(k) = \hat{a}(k)\,\hat{B}(k),$$

which is exactly the structure of optical imaging modulation transfer functions, radar matched filtering, and holographic reconstruction for shift-invariant systems. Thus, Fourier-domain transfer functions are one instance of semantic transfer functions—those that arise when the operator is shift-invariant—while the general semantic transfer function $\hat{T}(k)$ applies to any diagonalizable operator, enabling semantic phasors in arbitrary learned or conceptual bases.[13]

3.3 Semantic holography

3.3d Semantic holographic operators

Semantic holography arises from the interaction of two semantic fields, typically a concept C and an observed meaning B—through their spectral components. In the spectral-mode framework, holography is expressed directly in terms of the eigenvectors and eigenvalues of the semantic operator, generalizing optical and radar holography in which interference patterns are formed by multiplying spectral components in the Fourier basis. Let $\{v_k\}$ be the eigenvectors of a diagonalizable semantic operator T, and let:

$$\hat{B}(k) = \langle v_k, B\rangle, \hat{C}(k) = \langle v_k, C\rangle$$

be the spectral representations of the semantic fields $B$and C; these coefficients encode the amplitude and phase of each semantic mode.

3.3e Semantic hologram

The semantic hologram of $B$with respect to concept C is defined as the spectral interference pattern

$$\hat{H}_{C,B}(k) = \hat{C}(\bar{k})\,\hat{B}(k),$$

where complex conjugation implements semantic phase reversal. In optical holography, the hologram is formed by the interference term between a reference wave R and an object wave O, which involves products of the form $\bar{R}(k)\ O(k)$ in the spectral domain or $\bar{R}(x)\ O(x)$ in the spatial domain; this structure underlies the encoding of amplitude and phase in the hologram. The semantic expression captures the same structure, with the replacements spatial frequency → semantic mode, optical fields → semantic spectra, and complex conjugation → semantic phase reversal; the product $\bar{\hat{C}}(k)\ \hat{B}(k)$ encodes both amplitude and phase relationships, producing a distributed interference pattern across semantic modes.

Transforming back to the semantic domain yields

$$H_C(B) = \sum_k \bar{\hat{C}}(k)\ \hat{B}(k)\, v_k.$$

This hologram stores semantic information in a way that is distributed across modes, phase-preserving, robust to noise and partial occlusion, and invertible when illuminated by the originating concept, paralleling distributed holographic memories and HRR-style associative memories.

3.3f Semantic reconstruction

Reconstruction is achieved by illuminating the hologram with the concept C. In the spectral basis, this corresponds to multiplying the hologram by the reference wave $\hat{C}(k)$:

$$\hat{R}_{C,B}(k) = \hat{C}(k)\,\hat{H}_{C,B}(k) = |\ \hat{C}(k)\ |^2\ \ \hat{B}(k),$$

so that

$$R_C(B) = \sum_k |\ \hat{C}(k)\ |^2\ \ \hat{B}(k)\, v_k. [][]$$

This operation preserves semantic phase because $|\ \hat{C}(k)\ |^2$ is real and non-negative, and amplifies concept-aligned modes where $|\ \hat{C}(k)\ |$ is large, while suppressing irrelevant or noisy modes where $|\ \hat{C}(k)\ |$ is small, and reconstructs latent meaning as the semantic analogue of optical replay. The structure is mathematically identical to optical holography, where illuminating a hologram with the reference wave reconstructs the original object field.[14]

3.3g Interpretation in semantic terms

The semantic hologram $H_C(B)$ encodes the relational structure between concept and meaning where amplitude interactions reflect semantic salience alignment, phase interactions reflect relational and contextual alignment, and cross-mode interference captures distributed semantic structure, echoing the interpretation of superposed and bound vectors in HRR and related models.

Reconstruction $R_C(B)$ recovers latent meaning by reinforcing modes coherent with the concept and attenuating those that are not, providing a principled mechanism for semantic memory, semantic retrieval, context-dependent reconstruction, and distributed representation of meaning.[15]

3.3h Fourier-domain holography as a special case

When the semantic operator is convolutional and the diagonalizing basis is Fourier, $\hat{B}(k)$and $\hat{C}(k)$ become classical Fourier transforms. The hologram becomes:

$$H_{C,B}(x) = \mathcal{F}^{-1}\{\overline{\hat{C}(k)}\,\hat{B}(k)\}(x),$$

and reconstruction becomes

$$R_{C,B}(x) = \mathcal{F}^{-1}\{|\,\hat{C}(k)\,|^2\ \hat{B}(k)\}(x).$$

Thus, optical and radar holography appear as special cases of semantic holography, arising when the semantic operator is shift-invariant.[16]

3.3i Consequences for semantic systems

Semantic holography provides distributed memory that stores meaning across modes, phase-preserving reconstruction of latent semantic structure, robustness to noise, occlusion, and partial information, interpretability through spectral analysis, and compatibility with semantic phasor transformer architectures and vector-symbolic systems. This operator-theoretic formulation unifies semantic cognition with the mathematics of optical and radar holography while extending beyond the limitations of purely Fourier-centric models.[17]

4. Semantic Resolution Limits

4.1 Semantic resolution and estimation

Semantic resolution concerns the ability of a semantic system to distinguish nearby meanings under noise, ambiguity, and conceptual filtering. In the spectral-mode framework, resolution is governed by the eigenvalues and eigenvectors of the semantic operator associated with a concept or task. These eigenpairs define the semantic modes that are amplified, suppressed, or phase-shifted during inference, and resolution limits arise when two meanings become indistinguishable after being filtered through the spectral structure of the operator.

Let T_A be the semantic operator associated with concept A, with spectral decomposition

$$T_A = \sum_k \lambda_k P_k,$$

where $\lambda_k = r_k e^{i\theta_k}$ are complex eigenvalues and P_k are orthogonal projectors onto the semantic modes v_k. The spectral representation of a semantic field B is $\hat{B}(k) = \langle v_k, B \rangle$. Under this representation,

$$T_A B = \sum_k \lambda_k \hat{B}(k) v_k,$$

and semantic resolution is determined by the spectral weighting $|\lambda_k|$ and the phase response $\arg(\lambda_k)$.

4.2 Distinguishability and semantic distance

Two semantic hypotheses B_1 and B_2 are distinguishable under concept A if their filtered representations remain separated in the semantic Hilbert space:

$$d_A(B_1, B_2) = \| T_A B_1 - T_A B_2 \| > \varepsilon,$$

for some discrimination threshold $\varepsilon > 0$. Using the spectral decomposition of T_A,

$$T_A B_1 - T_A B_2 = \sum_k \lambda_k (\hat{B}_1(k) - \hat{B}_2(k)) v_k.$$

By Parseval's identity,

$$d_A^2(B_1, B_2) = \sum_k |\lambda_k|^2 \ |\hat{B}_1(k) - \hat{B}_2(k)|^2. [][]$$

This expression shows that modes with large $|\lambda_k|$ dominate discriminability, modes with small $|\lambda_k|$ are suppressed, and phase coherence across modes affects interference and alignment, directly paralleling modulation transfer-function analyses of resolution in optics and matched-filter responses in radar.

4.3 Rayleigh-like limit

The classical Rayleigh limit states that two optical sources are resolvable when their point spread functions overlap only to a specific degree, as quantified by the system's modulation transfer function and diffraction physics. In the semantic setting, the analogous condition is that the filtered semantic fields remain sufficiently separated in the spectral domain. Let $B(\theta)$ be a semantic field parameterized by a semantic variable θ(e.g., intent, role, topic). The semantic Rayleigh-like limit is the smallest $\Delta\theta$ such that

$$d_A(B(\theta), B(\theta + \Delta\theta)) > \varepsilon.$$

Using the spectral form,

$$d_A^2(\theta, \theta + \Delta\theta) = \sum_k |\lambda_k|^2 \ |\hat{B}(\theta, k) - \hat{B}(\theta + \Delta\theta, k)|^2.$$

Resolution improves when $|\lambda_k|$ remains large on high frequency semantic modes, arg (λ_k)varies smoothly across modes, and the operator is well-conditioned on its spectral support, in analogy with how optical diffraction limits resolution via the MTF's spectral cutoff.

4.4 Semantic Cramér–Rao bound

Let θbe a semantic parameter to be estimated from a noisy semantic field

$$B_{\text{obs}}(x) = B(\theta, x) + N(x),$$

where $N$is semantic noise with spectral density $S_N(k)$. In the spectral basis, the Fisher information under concept A.

$$I_A(\theta) = \sum_k \frac{|\lambda_k|^2 \ |\partial_\theta \hat{B}(\theta, k)|^2}{S_N(k)},$$

and the semantic Cramér–Rao bound is:

$$\text{Var}\,(\hat{\theta}) \geq \frac{1}{I_A(\theta)}. [][]$$

This bound shows that modes with large $|\lambda_k|$and strong parameter sensitivity $|\partial_\theta \hat{B}(\theta, k)|$contribute most to estimation, while modes dominated by noise contribute little; semantic resolution is fundamentally limited by the spectral structure of both the operator and the noise, paralleling Cramér–Rao analyses in radar and statistical signal processing.

4.5 Semantic ambiguity function

The semantic ambiguity function measures the similarity between two filtered semantic fields separated by a semantic parameter shift $\Delta\theta$, closely paralleling the radar ambiguity function and optical coherence functions.

$$\chi_A(\Delta\theta) = \frac{|\langle T_A B(\theta), T_A B(\theta+\Delta\theta)\rangle|}{\| T_A B(\theta) \| \ \| T_A B(\theta+\Delta\theta) \|}.$$

Equivalently, in the spectral representation,

$$\chi_A(\Delta\theta) = \frac{|\sum_k |\lambda_k|^2 \ \hat{B}(\theta,k)\, \overline{\hat{B}(\theta+\Delta\theta,k)}\,|}{(\sum_k |\lambda_k|^2 \ |\hat{B}(\theta,k)|^2)^{1/2} (\sum_k |\lambda_k|^2 \ |\hat{B}(\theta+\Delta\theta,k)|^2)^{1/2}}.$$

This is the semantic analogue of the optical coherence function, the radar ambiguity function, and the matched-filter correlation function, which quantify how well shifted or Doppler-shifted signals can be distinguished.

4.6 Interpretation and consequences

The spectral-mode formulation yields several insights: semantic resolution is governed by the spectral support of $|\lambda_k|$; semantic ambiguity arises when two meanings differ only in modes that are strongly suppressed; and semantic signal-to-noise ratio improves when the operator amplifies modes where signal structure differs from noise. Semantic limits arise from bandwidth, phase coherence, and operator conditioning, and—at the level of the mathematics—these limits match those of optics and radar because they derive from the same spectral geometry of diagonalizable operators acting on complex fields.

The semantic Fisher information measures how sensitively a semantic field changes with respect to a parameter—such as a connotation, nuance, or contextual shift—while the semantic Cramér–Rao bound establishes the fundamental limit on how precisely such semantic parameters can be inferred in the presence of semantic noise. Likewise, the semantic ambiguity function characterizes how meaning becomes uncertain under displacement or relational transformation, providing a spectral measure of how denotations (stable content) and connotations (context-dependent relational meaning) interfere, overlap, or blur, analogous to classical ambiguity and coherence functions. Together, these constructs show that semantic phasors support principled estimation, uncertainty quantification, and resolution analysis, enabling practical semantic inference systems analogous to those used in radar, optics, and communication technologies.

5. Unification with optics, radar, and holography

The operator-theoretic formulation of semantic inference reveals a structural unity across optical imaging, radar sensing, holographic reconstruction, and semantic cognition: each manipulates complex fields through linear operators that become diagonal in an appropriate spectral basis. The eigenvalues encode gain and phase rotation, while the eigenvectors define modes along which information evolves independently, forming the foundation of the proposed unification.

In optics and radar, the diagonalizing basis is typically Fourier because the underlying propagation and filtering operators are convolutional; in holography, interference patterns arise from the products of spectral components; in semantics, the diagonalizing basis is the eigenbasis of the semantic operator. In all cases, complex phasor fields are transformed by diagonalizable operators, and the governing mathematics is identical up to the choice of basis and physical interpretation.

5.1 Optical imaging and computing

An optical imaging system maps an input field $B(x)$to an output field $TB(x)$ through a linear operator determined by the system's point spread function; for shift-invariant systems this operator is convolutional and diagonalized by the Fourier transform. In the Fourier basis, the system acts as a spectral multiplier:

$$\hat{B}_{\text{out}}(k) = \hat{T}(k)\,\hat{B}(k),$$

where $\hat{T}(k)$is the optical modulation transfer function that determines which spatial modes are transmitted, suppressed, or phase-shifted, thereby governing resolution, contrast, and aberrations. In the semantic setting, a concept acts as a semantic lens with a transfer function $\hat{T}(k)$ defined in its own spectral basis, so semantic resolution, semantic blur, and semantic aberrations arise from the same spectral mechanisms.

5.2 Radar sensing and matched filtering

Radar systems transmit a waveform, receive an echo, and apply a matched filter to maximize constructive interference with a hypothesized target. The matched filter is a linear operator that becomes diagonal in the Fourier (or time–frequency) basis, yielding:

$$\hat{R}(k) = \overline{\hat{C}(k)}\,\hat{B}(k),$$

where $C$is the transmitted waveform. Ambiguity functions, resolution limits, and SNR are determined by the spectral structure of $\hat{C}(k)$. In the semantic domain, a concept or task plays the role of the transmitted waveform: semantic inference becomes a form of semantic radar, in which the concept defines a spectral filter that highlights certain semantic modes and suppresses others, and semantic ambiguity and resolution follow from the operator's spectral geometry.

5.3 Holography and interference-based reconstruction

A hologram encodes the interference pattern between an object field $B$and a reference field C; in the spectral domain the hologram takes the form $H(k) = \overline{\hat{C}(k)}\,\hat{B}(k)$, and reconstruction is achieved by illuminating the hologram with the reference wave, yielding $R(k) = |\hat{C}(k)|^2 \ \hat{B}(k)$. This process preserves phase information and enables three-dimensional reconstruction, partial reconstruction, and robustness to occlusion. In the semantic setting, the same structure yields semantic holography: a concept C generates a hologram of a meaning field B via their spectral interference pattern, and reconstruction recovers latent meaning when the hologram is illuminated by the originating concept, providing a distributed memory mechanism closely related to HRR-style holographic reduced representations.

5.4 Semantic inference as a spectral operator system

Semantic inference is governed by the same class of operators as optics, radar, and holography: diagonalizable linear operators acting on complex fields. In the eigenbasis of a semantic operator, inference takes the form:

$$\hat{B}_{\text{out}}(k) = \lambda_k \, \hat{B}(k),$$

where $\lambda_k = r_k e^{i\theta_k}$ encodes semantic gain and phase rotation; modes with large $|\lambda_k|$ dominate interpretation, while modes with small $|\lambda_k|$ are suppressed. Semantic ambiguity, semantic resolution, and semantic SNR all follow directly from the spectral structure of λ_k, just as in optical and radar systems.

5.5 Consequences of the unification

This unification implies that semantic resolution limits follow the same mathematics as optical diffraction limits and radar ambiguity functions, that semantic SNR is governed by spectral overlap between semantic modes and conceptual filters, and that semantic aberrations arise from irregular phase responses in the semantic transfer function. Semantic holography offers distributed, phase-preserving memory with resolution and capacity properties analogous to holographic reduced representations. Semantic phasor transformers implement these operators computationally through learned spectral bases and diagonal spectral filters, making meaning, light, radar echoes, and holographic patterns instances of a single underlying structure: complex fields evolving under spectral operators.

6. Architectural embodiment

6.0 Overview

The semantic phasor framework admits a direct computational realization. The key idea is that semantic operators are most naturally expressed in a spectral basis, where they act as diagonal complex multipliers. The Semantic Phasor Transformer Block implements this structure by learning a complex embedding space, learning a spectral basis in which semantic operators are diagonal, and performing semantic filtering and holographic interference in that basis.[18]

6.1 Discretization of the semantic field

Let $\Omega_d \subset \mathbb{R}^d$ be a finite sampling grid. A continuous semantic field $B: \mathbb{R}^d \to \mathbb{C}$ is represented by the vector of its sampled values on a finite grid $B = (B(x_1), \ldots, B(x_n))^\top \in \mathbb{C}^n$, exactly as continuous fields are discretized in imaging, numerical PDE solvers, and neural operator methods.[19]

A learned unitary matrix $U \in \mathbb{C}^{n\times n}$ serves as the semantic spectral transform, generalizing the discrete Fourier transform.micro.magnet.fsu+1 The requirement that U is unitary ensures energy preservation $\| B \|_2 = \| UB \|_2$, invertibility $U^{-1} = U^*$, and orthogonality of semantic modes, by Parseval's identity for unitary transforms.[20]

When U = F (the DFT matrix), this reduces to the Fourier case; when U is learned, the model discovers its own semantic mode basis, analogous to eigenmode decompositions in wave and communication systems.[21]

6.2 Complex embeddings and semantic phasor tokens

Let x_t denote the input token representation. A learned complex embedding matrix W_{phasor} maps tokens into the complex semantic space:

$$z_t = W_{\text{phasor}} x_t \in \mathbb{C}^n, (z_t)_k = r_{t,k} e^{i\phi_{t,k}}.$$

Each component $(z_t)_k = r_{t,k} e^{i\phi_{t,k}}$ is a semantic phasor, with magnitude encoding semantic strength and phase encoding relational orientation, in line with complex-valued embedding models where relations act as rotations in phase.[22]

This matches the physics of phasors and the mathematics of complex embeddings used in complex-valued RNNs, holographic reduced representations (HRRs), Fourier-feature networks, complex attention mechanisms, and wave-based neural architectures.[23]

Real-valued embeddings primarily encode magnitude-like information, whereas complex embeddings can additionally encode relational structure in phase, which has been exploited in complex knowledge-graph embeddings and HRR-style binding/unbinding.

6.3 Spectral transfer heads

Each attention head implements a diagonal operator in a learned spectral basis. For head h, let

$$A_h = \text{diag}(a_h(1), \ldots, a_h(n))$$

be a learned diagonal matrix of complex eigenvalues. The head output is $y_{t,h} = U^{-1} A_h U z_t$, a unitary similarity transform that is structurally identical to how normal operators are diagonalized in modal bases for waves, optics, and linear time-invariant filters.[24]

Each $a_h(k) = \rho_{h,k} e^{i\psi_{h,k}}$ encodes gain and phase delay for mode k, directly generalizing Fourier-domain filtering where transfer functions act as complex multipliers on each spectral component.[25]

This is a unitary similarity transform, structurally identical to the representation of quantum observables in the Heisenberg picture, linear time-invariant filters in Fourier space, and wave scattering operators in a modal basis.[26]

Then:

The transform performs spectral decomposition $\hat{z}_t = U z_t$, diagonal spectral filtering $A_h \hat{z}_t$, and reconstruction $U^{-1} A_h U z_t$. Each coefficient

$$a_h(k) = \rho_{h,k} e^{i\psi_{h,k}}$$

encodes semantic gain $\rho_{h,k}$ and phase delay/rotation $\psi_{h,k}$ for semantic mode k, generalizing Fourier-domain filtering to arbitrary learned semantic modes.[27]

6.4 Holographic Memory

The semantic hologram is $H_t = U^{-1}(\bar{\hat{z}}_c \odot \hat{z}_t)$, where $\bar{\hat{z}}_c$ is the complex conjugate of the concept spectrum, matching the interference term between reference and object waves in optical holography and where $\odot$ denotes elementwise multiplication and $\bar{\hat{z}}_c$ as the complex conjugate of the concept spectrum.[28,29]

Reconstruction $r_t = U^{-1}(|\hat{z}_c|^2 \odot \hat{z}_t)$ is directly analogous to illuminating a hologram with the reference wave to replay the object field, and to HRR-style holographic retrieval in vector-symbolic memory.[30] As in optical holography, this yields phase-preserving semantic memory, distributed storage, and robust reconstruction of latent meaning from partial or noisy data, closely paralleling HRR-style holographic memories.

6.5 Full Semantic Phasor Transformer Block

A complete block consists of multiple spectral transfer heads, holographic memory output, concatenation, projection back into the complex embedding space, and residual connections with normalization, mirroring standard transformer block design.[31]

At the operator level, the block implements a learned spectral operator:

$$T = U^{-1} D U,$$

where D is block-diagonal with learned complex eigenvalues (combining all heads and holographic channels).

This is the canonical form of any normal (unitarily diagonalizable) linear operator, any spectral filter, any quantum Hamiltonian in diagonal form, and any wave propagation operator in modal coordinates. math.[32]

In this realization:

- semantic modes correspond to columns of U;
- semantic phasor eigenvalues correspond to diagonal entries of D;
- semantic inference is spectral filtering; and
- semantic memory is holographic interference in a learned spectral basis.[33]

7. Discussion and advantages

7.1 Resolution and discriminability

Semantic resolution is determined by the spectral structure of the semantic transfer function $\hat{T}(k)$: modes with large $|\hat{T}(k)|$ contribute strongly to semantic distinctions, while modes with small $|\hat{T}(k)|$ are suppressed, yielding semantic analogues of the Rayleigh limit in optics, the ambiguity function in radar, and the Cramér–Rao bound in estimation.

7.2 Phase-preserving meaning

- phase as relational/rotational info via complex embeddings.
- phase coherence as crucial in optical/holographic

Because semantic fields are complex valued, phase becomes a first-class representational dimension. Semantic phase encodes relational and contextual structure, and semantic operators rotate this phase through their eigenvalues. Phase coherence across semantic modes enables:

- interference-based meaning composition,
- holographic memory,
- reconstruction of latent semantic structure,
- robust discrimination under noise.

This role of phase directly parallels its function in optical and holographic systems.

7.3 Holographic memory and distributed representation

The semantic hologram

$$H_C(B) = \sum_k \overline{\hat{C}}(k)\ \hat{B}(k)\, v_k$$

stores meaning in a distributed interference pattern, and reconstruction

$$R_C(B) = \sum_k |\,\hat{C}(k)\,|^2\ \hat{B}(k)\, v_k$$

recovers latent semantic content when illuminated by the originating concept, paralleling optical replay and HRR-style associative memories.[34]

This mechanism provides distributed storage, robustness to noise and occlusion, and partial reconstruction from incomplete data, in line with holographic reduced representations and related holographic memory models.[35]

7.4 Compatibility with neural architectures

The Semantic Phasor Transformer Block is structurally compatible with transformer encoders: it uses a learned unitary matrix U to define the semantic spectral basis, diagonal matrices to encode semantic phasor eigenvalues, spectral filtering and holographic interference in that basis, and complex embeddings that preserve semantic phase, while retaining residual and normalization layers known from conventional transformer architectures.[36]

7.5 Fundamental limits

As in optics and radar, semantic resolution and effective semantic signal to noise ratio are bounded by:

- the bandwidth and conditioning of the semantic transfer function,
- the coherence of its phase response,
- the geometry of the semantic space,
- the spectrum of semantic noise, including ambiguity, polysemy, and contextual drift.

These limits clarify where semantic phasor methods provide the greatest leverage and where semantic distinctions become irrecoverable.

The spectral-mode framework shows that semantic inference, optical imaging, radar sensing, and holography are all governed by diagonalizable linear operators acting on complex fields; the novelty here is reinterpreting semantic content itself as a coherent phasor field subject to interference, filtering, and reconstruction, rather than proposing new physics.

8. Toy Example: A 2D Semantic Operator

Major Elements:

A [Semantic Domain

B [Semantic Operator

C [Spectral Decomposition

D [Semantic Phasors

E [Semantic Transfer Function

F [Reconstruction / Meaning

8.1 A simple 2D semantic operator

Consider the 2×2 matrix which represents the semantic operator $\hat{S}$.

$$W = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}.$$

The Spectral Decomposition $\{\phi_k\}, \{\lambda_k\}$ of operator $\hat{S}$ can be viewed as a combination of rotation and scaling in a two-dimensional semantic plane. Its eigenvalues are

$$\lambda_{1,2} = 1 \pm i = \sqrt{2}\, e^{\pm i\pi/4},$$

so both modes have gain $\sqrt{2}$and phases $\pm\pi/4$.

The corresponding (unnormalized) eigenvectors are

$$v_1 = \begin{bmatrix} 1 \\ -i \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ i \end{bmatrix}.$$

Thus, W has two semantic phasor modes:

- mode 1: gain $\sqrt{2}$, phase $+\pi/4$(eigenpair (λ_1, v_1)),
- mode 2: gain $\sqrt{2}$, phase $-\pi/4$(eigenpair (λ_2, v_2)).

Intuitively, each mode represents a complex "direction" in which meanings evolve independently under the action of W.

8.2 Toy "tokens" as 2D concepts

Now define four toy tokens as two-dimensional real vectors $t_A, t_B, t_C, t_D \in \mathbb{R}^2 \subset \mathcal{H}$ as representing the semantic signals $f(x)$:

$$t_A = \begin{bmatrix}1\\0\end{bmatrix}, t_B = \begin{bmatrix}0\\1\end{bmatrix}, t_C = \begin{bmatrix}1\\1\end{bmatrix}, t_D = \begin{bmatrix}1\\-1\end{bmatrix}.$$

These represent four simple "concepts" embedded in a 2D semantic-domain plane: t_A and t_B are axis-aligned; t_C and t_D lie along the diagonals.

8.3 Decomposing tokens into semantic modes

Normalize the eigenvectors to obtain an orthonormal basis:

$$\hat{v}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}, \hat{v}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}.$$

Decomposing tokens into semantic modes computes the inner product:

$$c_k = \langle \hat{v}_k, t \rangle,$$

where $\langle u, v \rangle = u^* v$is the standard inner product on $\mathbb{C}^2$.

Token A

For $t_A = [1,0]^\top$,

$$c_1^A = \langle \hat{v}_1, t_A \rangle = \frac{1}{\sqrt{2}}[1 \quad i]\begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{\sqrt{2}},$$
$$c_2^A = \langle \hat{v}_2, t_A \rangle = \frac{1}{\sqrt{2}}[1 \quad -i]\begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}.$$

Token A is therefore an equal-magnitude superposition of both semantic modes with the same phase.

Token B

For $t_B = [0,1]^\top$,

$$c_1^B = \langle \hat{v}_1, t_B \rangle = \frac{1}{\sqrt{2}}[1 \quad i]\begin{bmatrix}0\\1\end{bmatrix} = \frac{i}{\sqrt{2}},$$

$$c_2^B = \langle \hat{v}_2, t_B \rangle = \frac{1}{\sqrt{2}}[1 \quad -i]\begin{bmatrix}0\\1\end{bmatrix} = -\frac{i}{\sqrt{2}}.$$

Here, the magnitudes | c_1^B | and | c_2^B | are equal, but their phases differ by π (they are negatives of each other). Tokens $C$and D can be analyzed in the same way: each token is a particular superposition of the two semantic modes, characterized by its complex coefficients (c_1, c_2).

Reconstruction will result from $f'(x) = \sum_k c_k' \, \phi_k(x)$ where

- a reconstruction of Wt_Aor Wt_B uses the spectral formula
- or a reconstruction after applying a simple transfer function.

8.4 Applying a semantic transfer function and reconstructing tokens

To complete the semantic-phasor pipeline, we now apply a simple semantic transfer function in the spectral domain and reconstruct the transformed tokens back in the original 2D semantic plane.

Transfer function as one application of W. For this toy example, take the semantic transfer function to be "apply the operator once" in the eigenbasis, i.e.

$$H(\lambda_k) = \lambda_k, c_k' = H(\lambda_k)c_k = \lambda_k c_k.$$

This corresponds exactly to acting with W on a token:

$$t' = Wt.$$

Token A. Recall:

$$t_A = \begin{bmatrix}1\\0\end{bmatrix}, \hat{v}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}, \hat{v}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix},$$

$$\lambda_1 = 1 + i, \lambda_2 = 1 - i,$$

and the phasor coefficients for t_A are

$$c_1^A = \langle \hat{v}_1, t_A \rangle = \frac{1}{\sqrt{2}}, c_2^A = \langle \hat{v}_2, t_A \rangle = \frac{1}{\sqrt{2}}.$$

Applying the transfer function:

$$c_1^{A'} = \lambda_1 c_1^A = (1+i)\frac{1}{\sqrt{2}}, c_2^{A'} = \lambda_2 c_2^A = (1-i)\frac{1}{\sqrt{2}}.$$

Reconstruct the transformed token:

$$t_A' = \sum_{k=1}^{2} c_k^{A'}\, \hat{v}_k = c_1^{A'}\hat{v}_1 + c_2^{A'}\hat{v}_2.$$

Substituting:

$$t_A' = (1+i)\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix} + (1-i)\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix} = \frac{1}{2}\left[(1+i)\begin{bmatrix}1\\-i\end{bmatrix} + (1-i)\begin{bmatrix}1\\i\end{bmatrix}\right].$$

Compute each term:

$$(1+i)\begin{bmatrix}1\\-i\end{bmatrix} = \begin{bmatrix}1+i\\-i-i^2\end{bmatrix} = \begin{bmatrix}1+i\\-i+1\end{bmatrix},$$

$$(1-i)\begin{bmatrix}1\\i\end{bmatrix} = \begin{bmatrix}1-i\\i-i^2\end{bmatrix} = \begin{bmatrix}1-i\\i+1\end{bmatrix}.$$

Adding:

$$\begin{bmatrix}1+i\\-i+1\end{bmatrix} + \begin{bmatrix}1-i\\i+1\end{bmatrix} = \begin{bmatrix}2\\2\end{bmatrix}.$$

Thus:

$$t_A' = \frac{1}{2}\begin{bmatrix}2\\2\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}.$$

Directly applying W to t_A gives:

$$Wt_A = \begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix},$$

which matches the reconstruction from the transformed phasors. This explicitly shows that:

- applying the operator in the spectral domain via $c_k' = \lambda_k c_k$, and
- reconstructing via $t' = \sum_k c_k'\, \hat{v}_k$,

is equivalent to applying W in the original semantic plane.

Interpretation. In this toy model, the semantic transfer function $H(\lambda_k) = \lambda_k$rotates and scales each semantic phasor according to its eigenvalue. The reconstructed token t_A' is a new "concept" whose meaning arises from the interference of the rotated phasors. This completes the A→F pipeline in concrete form:

- A: token t_A

The overall pattern is:

- some tokens excite both modes with similar magnitudes but different relative phases;
- under repeated application of W, each token evolves into a characteristic interference pattern of the two semantic phasors.

In this toy model, "concepts" are therefore distinguished by the semantic modes they excite and by the complex coefficients with which they do so.

9. Turning Semantic Phasors into a Design Principle

The toy model suggests how to build architectures where phasor structure is *designed in* rather than arising accidentally.

9.1 Complex-valued representations

- Represent hidden states as complex vectors $h \in \mathbb{C}^d$.
- Implement linear layers as complex matrices $W \in \mathbb{C}^{d \times d}$.
- Apply nonlinearities to magnitudes, to real and imaginary parts separately, or using holomorphic constraints when appropriate.

This makes phase a first-class representational quantity: every dimension carries an explicit angle, not just a magnitude.

9.2 Approximately normal operators

To enable stable and interpretable spectral analysis, you want operators that are (approximately) normal:

- Parameterize W as

  $$W = UDU^*,$$

  where U is unitary and D is diagonal with entries $\rho_k e^{i\phi_k}$.

- Then:
  - the eigenvectors are the columns of U,
  - the eigenvalues are the diagonal entries of D,

- semantic phasor modes are explicitly built into the architecture.

If an exact factorization is too restrictive, you can enforce this structure softly with a regularization term

$$\| WW^* - W^*W \|^2,$$

which encourages W to be approximately normal (and thus well behaved spectrally).

9.3 Real-valued implementations via 2×2 blocks

If the overall model must remain real-valued, we can still encode complex phasors:

- pair coordinates (h_{2k}, h_{2k+1})as the real and imaginary parts of one complex component;
- constrain each 2×2block of the weight matrix to approximate a rotation plus scaling:

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix}.$$

This yields a real-valued implementation of complex phasors while preserving the underlying geometric structure.

10. Where Semantic Phase Lives in a Token

10.1 Phase in complex embeddings

With complex embeddings, a token can be written as

$$e_{\text{token}} = [x_1 e^{i\phi_1}, x_2 e^{i\phi_2},\ \ldots, x_d e^{i\phi_d}],$$

where:

- magnitudes $x_k$encode the strength of each semantic feature;
- phases $\phi_k$encode the token's relational orientation along that feature.

For two tokens, the relative phase along dimension k,

$$\Delta\phi_k = \phi_k^{(\text{token 1})} - \phi_k^{(\text{token 2})},$$

can encode distinctions such as opposition vs alignment, grammatical role, discourse polarity, or other relational contrasts.

10.2 Phase in real-valued models

Even when the model is purely real-valued, you can interpret an implicit phase:

- treat each pair (h_{2k}, h_{2k+1}) as (Re , Im);
- define the phase of pair $k$as

$$\phi_k = \text{atan2}\,(h_{2k+1}, h_{2k}).$$

Here, phase is implicit in the geometry of the embedding but can be made explicit via this mapping.

10.3 Phase in semantic composition/ Reconstruction

When a token passes through a semantic operator A with eigenvalues

$$\lambda_k = \rho_k e^{i\theta_k},$$

and its projection onto eigenvector v_k has coefficient c_k, the transformed coefficient is

$$c_k' = \lambda_k c_k.$$

Thus:

- the amplitude $|\ c_k\ |$ is scaled by ρ_k;
- the phase $\arg(c_k)$ is shifted by θ_k.

Phase is therefore explicit both:

- in the complex coordinates of the token in the eigenbasis (the c_k), and
- in the arguments of inner products such as $\langle v_k, e_{\text{token}}\rangle$.

This is the core phasor-based picture: semantic modes are eigenvectors, and semantic phase lives in how tokens align with those modes and how operators rotate those phases.

11. Conclusion

This paper develops a rigorous operator-theoretic foundation for semantic representation and inference, unified with physical sensing systems, and derived transformer-like architecture as discretized embodiments. It establishes a mathematically principled path toward new semantic architectures. Each meaning admits a unique expansion within a spectral basis, with complex coefficients interpreted as semantic phasors whose magnitudes and phases capture the contribution and alignment of each mode.

Treating "semantic inference" itself explicitly as a diagonalizable linear operator on a complex Hilbert space whose eigenbasis defines "semantic modes," in direct mathematical analogy with optical modes is not standard in the existing literature. Frameworks in semantic communication and "semantic pointers" talk about semantic feature spaces, entropy, or category-theoretic structures, but not about a phasor-like spectral operator calculus unifying optics and semantics at this level of detail.[37] Thus, the operator-theoretic side is classical but the move to say "meaning, such as light, evolves via spectral amplification, suppression, and phase rotation in a common diagonalizable-operator formalism" is a conceptual unification.

Within the assumptions as stated, the mapping is mathematically coherent in that:

a. Modeling both optical fields and "semantic fields" as elements of complex Hilbert spaces is standard operator-theoretic practice; the non-standard step interpreting the latter as semantics. If a semantic operator T_A is normal or diagonalizable, then speaking of its eigenbasis as "semantic modes" and of its Fourier-like coefficients as "semantic phasors" is consistent with spectral theory.

b. The analogy "MTF ↔ semantic transfer function" is structurally sound: both are multiplicative symbols in a basis that diagonalizes the operator, controlling which modes are passed, attenuated, or phase-shifted.

Given explicit Hilbert spaces, operators, and spectral decompositions, the framework can be made rigorous; it aligns with standard spectral theorem machinery. We do not find prior work that frames semantics itself as a complex phasor field in a Hilbert space, and explicitly identifies semantic inference operators with the same diagonalizable-operator class as optical imaging. Related work tends to use probabilistic, information-theoretic, or categorical foundations instead.

Classical Fourier-based models treat optical imaging as a convolution operator diagonalized in a Fourier-based basis. Building on standard spectral theory, we generalize this to a broader class of diagonalizable operators on complex Hilbert spaces and introduce a semantic-phasor framework in which semantic inference is represented by analogous spectral operators on semantic fields. This yields a unified operator-theoretic view in which optical imaging, radar sensing, and semantic processing are all instances of diagonalizable spectral filters acting on complex fields.

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