The Operator Calculus of Semantic Phasors: Formal Foundations and Spectral Analysis

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Abstract

This paper develops the mathematical foundations of semantic phasor theory, a framework in which linguistic meaning is represented as a spectral phasor superposition in a complex Hilbert space. Building on prior work introducing the conceptual architecture of semantic phasors, we formalize the operator calculus that governs semantic transformation. We begin by establishing a structural isomorphism between semantic meaning and classical AC signal analysis, showing that meaning vectors correspond to waveforms, semantic eigenmodes correspond to Fourier modes, and semantic coefficients correspond to complex phasors. Using this isomorphism, we derive three canonical operators generated by a word vector: (i) a projector that performs semantic correlation, (ii) a convolutional semantic filter that smooths or reshapes meaning, and (iii) a semantic-phasor rotation operator that modulates phase structure across semantic frequencies. We analyze the spectral behavior of each operator and show that they form a unified triadic calculus isomorphic to correlation, convolution, and modulation in signal theory. Finally, we demonstrate how these operators give rise to semantic holography, in which meaning is encoded and reconstructed through phase-coherent interference patterns. This work provides the mathematical core of a spectral operator theory of meaning, offering a principled foundation for semantic memory, contextual transformation, and compositional reasoning.

Significance Statement

This work introduces a mathematically rigorous operator calculus for semantic meaning, grounded in a structural isomorphism between linguistic representations and AC signal phasors. By showing that words generate projector, convolutional, and phase-rotation operators—direct analogs of correlation, convolution, and modulation in signal theory—this framework provides a unified spectral account of semantic selection, shaping, and contextual transformation. The resulting theory explains meaning not as a static vector but as a distributed, interference-capable phasor field, enabling a holographic model of semantic memory and interpretation. This approach offers a principled foundation for interpretable, physically inspired models of meaning and opens new pathways for integrating linguistic semantics with operator theory, signal processing, and cognitive modeling.[1,2]

Keywords: Semantic phasor, Complex Hilbert space, Semantic eigenmodes, Spectral decomposition, Phasor superposition, Semantic projector, Semantic correlation, Convolutional semantic filter, Semantic convolution, Phasor rotation

1 Introduction

Understanding meaning remains one of the central challenges in artificial intelligence, linguistics, and cognitive science. Contemporary models represent words and sentences as high-dimensional vectors, yet the mathematical structure underlying these representations remains largely opaque. Neural architectures such as transformers manipulate these vectors through learned matrices, but the operations themselves lack a principled semantic interpretation. As a result, modern AI systems achieve high performance while offering limited insight into how meaning is represented, transformed, or composed.[3]

Semantic phasor theory proposes a different foundation. Rather than treating meaning as a static point in a vector space, it models meaning as a spectral phasor superposition in a complex Hilbert space. In this view, each word or concept decomposes into semantic eigenmodes, each carrying a complex amplitude and phase. Meaning becomes a dynamic, interference-capable structure, more akin to a waveform than to a coordinate vector, paralleling phasor decompositions in AC signal analysis.[4]

This paper develops the mathematical foundations of semantic phasor theory introduced in Carvalko (2026).[5] In that earlier work, meaning was modeled as a spectral phasor superposition, and the conceptual architecture of semantic eigenmodes, interference, and holographic encoding was established. What remained deferred was the operator calculus governing how meanings are selected, shaped, and transformed within this spectral framework. The present paper provides formal machinery. By deriving projector, convolutional, and phase-rotation operators directly from word vectors, we supply the operator-theoretic structure implicit in the original theory and establish a unified spectral calculus for semantic transformation.

The contributions of this paper are fourfold. First, we formalize the isomorphism between semantic phasors and AC signal phasors, establishing a shared Hilbert-space structure. Second, we derive three canonical operators generated by a word vector: a projector that performs semantic correlation, a convolutional semantic filter that smooths or reshapes meaning, and a semantic-phasor rotation operator that modulates phase structure across semantic frequencies. Third, we analyze the spectral behavior of each operator, showing that they form a unified triadic calculus isomorphic to correlation, convolution, and modulation in signal theory. Finally, we show how these operators give rise to semantic holography, a distributed, interference-based mechanism for encoding and reconstructing meaning.[6]

By grounding semantics in a mathematically coherent spectral operator theory, this work provides a principled foundation for contextual transformation, compositional reasoning, and semantic memory, and it connects symbolic, statistical, and signal-theoretic approaches to meaning.[7]

2 Semantic Meaning as a Spectral Phasor Superposition

Meaning is represented in a complex Hilbert space $\mathcal{H}$, whose elements correspond to words, phrases, sentences, or more abstract conceptual structures. Let $\{e_k\}_{k\in K}$ denote an orthonormal basis of semantic eigenmodes, representing fundamental semantic directions that span the space of possible meanings.[8]

Any meaning vector $m \in \mathcal{H}$ admits a spectral decomposition

$$m = \sum_k c_k \ e_k,$$

where $c_k \in \mathbb{C}$ are semantic phasor coefficients. Each coefficient can be written as

$$c_k = | \ c_k \ | \ e^{i \arg (c_k)},$$

with magnitude $| \ c_k \ |$ representing the strength of the semantic mode and phase $\arg (c_k)$ representing contextual orientation or relational nuance.[9]

The inner product between two meanings $m_1, m_2 \in \mathcal{H}$, with decompositions $m_\ell = \sum_k c_{\ell k} \, e_k$, is

$$\langle m_1, m_2 \rangle = \sum_k c_{1k} \ \bar{c_{2k}},$$

which generalizes cosine similarity to the complex domain. The phase structure of the coefficients allows interference effects, so meanings may reinforce, cancel, or modulate one another depending on their relative phases, a property central to semantic holography and contextual modulation.[10]

Semantic phasor theory thus treats meaning not as a static point but as a phasor field—a structured distribution of amplitude and phase across semantic “frequencies.” This perspective enables a rich operator calculus, developed below.[11]

3 Formal Isomorphism with AC Signal Analysis

AC signal analysis represents a time-varying waveform $V(t)$as a Fourier series

$$V(t) = \sum_k C(\,\omega_k) e^{i\omega_k t},$$

where the exponentials $e^{i\omega_k t}$form an orthogonal set of sinusoidal modes and $C(\omega_k) \in \mathbb{C}$are complex phasors. This is the standard phasor representation of AC signals in communication and signal-processing.[12]

We define a linear mapping Φbetween semantic and signal spaces by

$$\Phi(e_k) = e^{i\omega_k t}, \Phi(c_k) = C(\omega_k),$$

and extend it to all $m = \sum_k c_k\, e_k$via linearity:

$$\Phi \left(\sum_k c_k\, e_k \right) = \sum_k C(\,\omega_k) e^{i\omega_k t}.$$

This mapping preserves:

- linearity by construction,
- orthogonality, since $\{e_k\}$and $\left\{e^{i\omega_k t}\right\}$ are respective orthonormal bases under their inner products,[13]
- inner products, as

$$\langle m_1, m_2 \rangle_{\mathcal{H}} = \sum_k c_{1k}\, \bar{c}_{2k} = \sum_k C_1\,(\omega_k)\; C_2(\bar{\omega}_k) = \langle \Phi(m_1), \Phi(m_2) \rangle_{L^2},$$

- spectral decomposition and phasor geometry, since amplitudes and phases are preserved.[14]

Thus $\Phi$is a Hilbert-space isomorphism between the semantic space and a suitable signal space.[15]

Under this isomorphism, we have the following correspondence:

Semantic phasor theory	**AC signal theory**
meaning vector m	waveform $V(t)$[16]

Semantic phasor theory	**AC signal theory**
eigenmodes e_k	Fourier modes $e^{i\omega_k t}$[17]
coefficients c_k	phasors $C(\omega_k)$[18]
(	c_k
arg (c_k)	phase arg $(C(\omega_k))$[19]

This isomorphism legitimizes the importation of operator constructions from signal theory into semantic space. In particular:

- projectors correspond to correlation or matched filters,
- convolutional semantic filters correspond to convolution kernels,
- semantic-phasor rotations correspond to phase modulators.[20]

4 Semantic Operators Derived from Word Vectors

In classical signal analysis, a waveform can generate operators such as correlation filters, convolution kernels, and phase modulators that act on other signals. In semantic phasor theory, a word vector is not merely a point in a high-dimensional space; it is also a generator of operators that act on other meanings. This dual role mirrors the way a reference waveform can serve as both a signal and the kernel of a filter in signal processing.

Given a word vector $w \in \mathcal{H}$, we introduce three canonical operator constructions:

1. the projector operator P_w,
2. the convolutional semantic filter K_w,
3. the semantic-phasor rotation operator U_w.

These three operators form a semantic analog of correlation, convolution, and modulation in signal processing.[21]

4.1 Projector Operator: Semantic Correlation

Given a word vector $w \in \mathcal{H}$, the (rank-one) projector operator is defined as

$$P_w = | w\rangle\langle w |.$$

For any meaning vector $m \in \mathcal{H}$, the action is

$$P_w m = \langle w, m\rangle \, w.$$

This operator extracts the component of $m$aligned with w, suppressing all orthogonal semantic modes. It is Hermitian and idempotent:[22]

$$P_w^\dagger = P_w, P_w^2 = P_w,$$

which follows from standard properties of rank-one projectors in Hilbert spaces.[23]

Signal-theoretic analogy. The projector corresponds to a matched filter or correlation operator that measures the similarity between a signal and a reference waveform.

Semantic interpretation. The projector isolates the semantic contribution of a word, functioning as a mechanism for attention, emphasis, or contextual selection.

4.2 Convolutional Semantic Filter: Semantic Smoothing and Shaping

A word can also generate a convolution-like operator that smooths or reshapes meaning. Let $k_w(i,j)$be a similarity kernel derived from the projections of basis vectors onto w, and define the operator[24]

$$K_w = \sum_{i,j} k_w\,(i,j)\ \mid e_i\rangle\langle e_j \mid.$$

Acting on a meaning vector $m = \sum_j m(j)\ e_j$, this operator has components

$$(K_w m)(i) = \sum_j k_w\,(i,j)\, m(j),$$

which is a discrete convolution or smoothing operation over semantic indices.[25]

In the semantic Fourier basis, $K_w$is diagonal:

$$K_w e_k = \widehat{k_w}(\omega_k)\, e_k,$$

where $\widehat{k_w}(\omega_k)$denotes the spectral (Fourier-like) transform of the kernel with respect to the eigenbasis.[26]

Signal-theoretic analogy. This operator mimics convolution with a filter that smooths, sharpens, or shapes a waveform.[27]

Semantic interpretation. The convolutional operator performs semantic smoothing, disambiguation, or blending, capturing how certain words (e.g. “almost,” “slightly,” “because”) reshape the meaning of surrounding content.[28]

4.3 Semantic-Phasor Rotation Operator: Meaning Transformation

The most expressive operator generated by a word vector is the semantic-phasor rotation operator

$$U_w = \exp(iH_w),$$

where $H_w$is a Hermitian generator constructed from the word vector. A simple choice is [29]

$$H_w = \alpha \mid w\rangle\langle w \mid,$$

with $\alpha \in \mathbb{R}$, but more general constructions allow frequency-dependent phase shifts by making $H_w$diagonal in the eigenbasis with eigenvalues $\theta_k(w)$.[30]

Acting on a meaning vector $m = \sum_k c_k\, e_k$, we have

$$U_w m = \sum_k e^{i\theta_k(w)}\; c_k\, e_k.$$

Since $H_w$is Hermitian, $U_w$is unitary:

$$U_w^\dagger U_w = e^{-iH_w} e^{iH_w} = I,$$

so it preserves inner products and norms by standard functional calculus for self-adjoint operators.[31]

Signal-theoretic analogy. $U_w$is analogous to a phase modulator that shifts the phase of each frequency component of a signal without altering its magnitude spectrum.[32]

Semantic interpretation. The rotation operator captures contextual transformation—how words like “should,” “might,” “if,” “why,” or “not” reorient meaning by altering relational structure (phases) rather than magnitudes.[33]

5 Spectral Analysis of the Three Operators

Because semantic phasor theory is isomorphic to AC signal analysis, each operator admits a clean spectral characterization. This section analyzes the eigenvalues, eigenvectors, and spectral effects of the projector, convolutional filter, and phasor rotation operator. [34]

5.1 Spectrum of the Projector

Let $P_w = | w\rangle\langle w |$. Then

$$P_w w = w, P_w v = 0 \text{for all } v \perp w,$$

so the spectrum of $P_w$is

$$\text{spec}\,(P_w) = \{1,0\},$$

with eigenvalue 1corresponding to eigenvector $w$and eigenvalue $0$with multiplicity $\dim\,(\mathcal{H}) - 1$. [35]

In spectral form,

$$P_w = \sum_k \lambda_k \ | v_k\rangle\langle v_k |,$$

with $\lambda_k \in \{0,1\}$and $\{v_k\}$an orthonormal basis including w.[36]

Interpretation. The projector isolates a single semantic “frequency” mode and suppresses all others, acting as an ideal pass-through for the direction w.[37]

5.2 Spectrum of the Convolutional Semantic Filter

Convolution is diagonal in the Fourier basis: for classical signals, $\widehat{h * x}(\omega) = \hat{h}(\omega)\,\hat{x}(\omega)$. In the semantic setting, the convolutional operator $K_w$satisfies[38]

$$\widehat{K_w m}(\omega) = \widehat{k_w}(\omega)\,\widehat{m}(\omega),$$

where $\widehat{m}(\omega_k)$are the semantic spectral coefficients and $\widehat{k_w}(\omega_k)$are the spectral values of the kernel.[39]

Thus the eigenvalues of $K_w$in the semantic eigenbasis are

$$\lambda_k = \widehat{k_w}(\omega_k),$$

with eigenvectors e_k.[40]

Interpretation. The word acts as a frequency-dependent gain control on semantic modes, amplifying or attenuating specific spectral components of meaning.

5.3 Spectrum of the Semantic-Phasor Rotation Operator

Since $U_w = e^{iH_w}$ with a Hermitian H_w, and H_w is diagonalizable with real eigenvalues $\theta_k(w)$, we have

$$H_w e_k = \theta_k(w)\, e_k, U_w e_k = e^{i\theta_k(w)} e_k.$$

Therefore, the eigenvalues of U_w are complex phases on the unit circle:

$$\lambda_k = e^{i\theta_k(w)}, |\lambda_k| = 1.$$

Interpretation. The word induces a phase rotation on each semantic frequency, altering interference patterns while preserving magnitude spectra.[41]

6 Semantic Holography and Interference

The operator calculus developed above provides the mathematical foundation for semantic holography. In optical holography, information is stored not in amplitudes alone but in phase-coherent interference patterns. An analogous principle applies to semantic meaning when represented as phasor fields.[42]

6.1 Meaning as an Interference Pattern

Given two meanings $m_1, m_2 \in \mathcal{H}$, their superposition

$$m = m_1 + m_2$$

has squared norm

$$\| m \|^2 = \langle m, m \rangle = \| m_1 \|^2 + \| m_2 \|^2 + 2\Re\langle m_1, m_2 \rangle,$$

so in a complex spectral representation we can write componentwise

$$|m|^2 = |m_1|^2 + |m_2|^2 + 2\Re(m_1 \bar{m_2}),$$

where the cross-terms encode relational meaning through interference.[43]

Interpretation. Semantic nuance emerges from interference between phasor components, analogous to holographic interference fringes.

6.2 Operators as Holographic Lenses

Each operator plays a role analogous to an optical element acting on wavefronts in holography.[44]

- The projector $P_w$selects a reference beam by isolating the component aligned with w.
- The convolutional filter K_w shapes the interference envelope via frequency-dependent gain.
- The phasor rotation $U_w$shifts phase structure, altering reconstruction without changing magnitudes.

Thus, semantic operators act as holographic lenses that transform meaning by manipulating spectral interference.[45]

6.3 Reconstruction of Meaning

Given a stored semantic hologram represented by an operator $H$and a reference phasor (or meaning) r, reconstruction is

$$m_{\text{recon}} = H \cdot r,$$

where "·" denotes the action of the hologram operator on the reference.[46]

This mirrors optical holography, where illuminating a hologram with the reference beam reconstructs the original wavefront.[47]

6.4 Implications

Semantic holography yields:

- distributed encoding: meaning is stored across all spectral modes,
- robustness: partial information can reconstruct approximate wholes,
- context sensitivity: reference phasors modulate reconstruction,
- compositionality: interference patterns combine meaningfully.[48]

This provides a unified explanation for metaphor, ambiguity, contextual shifts, emotional nuance, and semantic blending as interference phenomena in a spectral operator space.[49]

7 Discussion

Semantic phasor theory reframes meaning as a spectral, interference-capable structure rather than a static point in a vector space. The operator calculus developed here demonstrates that semantic transformation can be understood through the same mathematical lens that governs classical signal processing and phasor analysis.[50]

First, the triadic operator calculus—projector, convolutional filter, and phasor rotation—provides a principled account of how words act on meanings. These operators correspond to correlation, convolution, and modulation in AC signal theory, implying that linguistic composition is spectrally structured rather than purely additive. Words select, shape, and rotate semantic modes, altering both amplitude and phase, which helps explain context sensitivity, natural ambiguity, and large interpretive differences caused by subtle phrasing changes.[51]

Second, the spectral analysis of these operators shows that semantic transformation is fundamentally frequency-dependent. Some words amplify high-frequency semantic modes (e.g. intensifiers), others suppress them (e.g. hedges), and still others rotate phase structure (e.g. modal verbs, negation, causal connectives). This frequency-dependent behavior parallels the operation of filters and modulators in signal processing and suggests that semantic cognition may rely on similar spectral principles.[52]

Third, semantic holography emerges naturally from phasor-based representation. Because meaning is encoded in distributed amplitude–phase patterns, interference plays a central role in semantic memory and reconstruction, providing a unified explanation for metaphor, analogy, emotional nuance, and contextual reinterpretation as constructive and destructive interference among semantic modes. The holographic perspective also explains robustness (partial cues can reconstruct approximate meanings) and flexibility (different reference phasors yield different contextual interpretations).[53]

Finally, the operator calculus offers a new lens for understanding modern neural architectures. Transformer attention mechanisms, convolutional layers, and phase-like positional encodings can be interpreted as approximations to the operators introduced here, at least at a coarse level. This suggests that semantic phasor theory may serve both as a mathematical model of meaning and as a theoretical foundation for interpretable, physically inspired AI systems.[54]

8 Conclusion

This paper has developed mathematical machinery underlying semantic phasor theory, establishing a unified operator calculus for meaning grounded in a structural isomorphism with AC signal analysis. By treating meaning as a spectral phasor superposition, we showed that a word vector can generate three fundamental operators—projector, convolutional filter, and phasor rotation—each with a clear spectral interpretation and a direct analog in classical signal processing. These operators form a triadic calculus that enables selection, shaping, and transformation of meaning through amplitude and phase manipulation of semantic eigenmodes.[55]

The resulting framework provides a principled explanation for contextual modulation, semantic blending, ambiguity resolution, and compositional reasoning, and it naturally supports a holographic view of semantic memory. Because meaning is stored and reconstructed through distributed phase-coherent patterns, the model predicts robustness and generalization properties that are desirable for artificial intelligence systems.[56]

Future work will extend this operator calculus to multi-word expressions, discourse-level dynamics, and multimodal meaning, and will explore its implications for neural architectures that implicitly perform spectral operations. By grounding semantics in a mathematically coherent spectral operator theory, this work opens a path toward unified, interpretable, and physically inspired models of meaning.[57]

Appendix A. Mathematical Details

A.1 Proof of the Semantic–Signal Isomorphism

Let $\mathcal{H}_S$and $\mathcal{H}_V$denote the semantic and signal Hilbert spaces, respectively. Assume $\{e_k\}$is an orthonormal basis in $\mathcal{H}_S$and $\{e^{i\omega_k t}\}$is an orthonormal basis in $\mathcal{H}_V$under the standard L^2inner product on one period or on $\mathbb{R}$with appropriate normalization.[58]

Define $\Phi: \mathcal{H}_S \to \mathcal{H}_V$via

$$\Phi(e_k) = e^{i\omega_k t}, \Phi \boxed{,} \left(\sum_k c_k \, e_k \right) = \sum_k C(\,\omega_k) e^{i\omega_k t},$$

where $C(\omega_k) = c_k$.[59]

Linearity follows immediately from the definition. To show that Φpreserves inner products, let $m_\ell = \sum_k c_{\ell k} \, e_k \in \mathcal{H}_S$for $\ell = 1,2$. Then [60]

$$\langle m_1, m_2\rangle_{\mathcal{H}_S} = \sum_k c_{1k}\, \overline{c_{2k}} = \sum_k C_1\,(\omega_k)\, C_2(\bar{\omega}_k) = \langle \Phi(m_1), \Phi(m_2)\rangle_{\mathcal{H}_V},$$

which shows that $\Phi$is an isometry. Surjectivity holds for the closed span of the modes $\{e^{i\omega_k t}\}$, so $\Phi$is a unitary isomorphism between $\mathcal{H}_S$and this subspace of $\mathcal{H}_V$.[61]

A.2 Spectrum of the Projector

Let $P_w = | w\rangle\langle w |$with $\| w \| = 1$, Then [62]

$$P_w w = \langle w, w\rangle w = w,$$

so $w$is an eigenvector with eigenvalue 1. For any $v \perp w$, we have $\langle w, v\rangle = 0$, hence[63]

$$P_w v = \langle w, v\rangle\, w = 0,$$

so every vector orthogonal to $w$is an eigenvector with eigenvalue 0. Therefore,[64]

$$\text{spec}\,(P_w) = \{0,1\}.$$

A.3 Diagonalization of the Convolutional Operator

Let $K_w$be defined by the kernel $k_w(i,j)$in the basis $\{e_k\}$:

$$K_w = \sum_{i,j} k_w\,(i,j)\ \mid e_i\rangle\langle e_j \mid.$$

Define a semantic Fourier transform mapping basis indices $k$to spectral indices ω_k, and denote by $\widehat{k_w}(\omega_k)$the transform of the kernel along this axis.[65]

For a basis vector e_k,

$$K_w e_k = \sum_{i,j} k_w\,(i,j)\ \mid e_i\rangle\langle e_j, e_k\rangle = \sum_i k_w\,(i,k)\, e_i.$$

In the spectral (Fourier) basis, this becomes

$$\widehat{K_w e_k}(\omega_\ell) = \widehat{k_w}(\omega_\ell)\,\widehat{e_k}(\omega_\ell) = \widehat{k_w}(\omega_k)\,\delta_{\ell k},$$

so

$$K_w e_k = \widehat{k_w}(\omega_k)\,e_k.$$

Thus each $e_k$is an eigenvector with eigenvalue $\lambda_k = \widehat{k_w}(\omega_k)$.[66]

A.4 Unitary Property of the Phasor Rotation Operator

Let $U_w = e^{iH_w}$ with H_w Hermitian on $\mathcal{H}$. The spectral theorem implies there exists an orthonormal basis of eigenvectors $\{e_k\}$with real eigenvalues $\theta_k(w)$such that

$$H_w = \sum_k \theta_k\,(w)\ \mid e_k\rangle\langle e_k \mid.$$

Then

$$U_w = e^{iH_w} = \sum_k e^{i\theta_k(w)}\ \mid e_k\rangle\langle e_k \mid.$$

Taking the adjoint,

$$U_w^\dagger = \sum_k e^{-i\theta_k(w)}\ \mid e_k\rangle\langle e_k \mid,$$

so

$$U_w^\dagger U_w = \sum_k \mid e_k\rangle\langle e_k \mid = I,$$

and $U_w$is unitary.[67]

A.5 Example: Action of All Three Operators on a Meaning Vector

Let

$$m = \sum_k c_k \, e_k.$$

Then:

- Projector:

$$P_w m = \langle w, m \rangle \, w.$$

- Convolutional filter (in the spectral basis):

$$K_w m = \sum_k \hat{k}_w \, (\omega_k) \, c_k \, e_k.$$

- Phasor rotation:

$$U_w m = \sum_k e^{i\theta_k(w)} \; c_k \, e_k.$$

These formulas summarize the triadic action of word-generated operators on a generic meaning vector.[68]

Appendix B. Toy Example: Projecting a Paragraph into the Hope–Despair Plane

This appendix illustrates how a short paragraph can be analyzed using the semantic phasor operator calculus in a low-dimensional emotional subspace.[69]

B.1 Paragraph and Representation

Consider the paragraph:

“She walked into the morning light unsure of what the day would bring, yet something in the quiet air felt like the beginning of a long-awaited turning.”

Let the paragraph embedding be

$$m = \sum_k c_k\, e_k.$$

We focus on two semantic eigenmodes:

- e_H: a “Hope” eigenmode,
- e_D: a “Despair” eigenmode.

Projecting $m$onto these modes gives

$$c_H = \langle e_H, m \rangle, c_D = \langle e_D, m \rangle.$$

For illustration, assume

$$c_H = 0.78\, e^{i\phi_H}, c_D = 0.22\, e^{i\phi_D},$$

with $| c_H | \gg | c_D |$.[70]

B.2 Projector Operators

Define the projectors

$$P_H = | e_H \rangle \langle e_H |, P_D = | e_D \rangle \langle e_D |.$$

The extracted components are

$$P_H m = c_H e_H, P_D m = c_D e_D.$$

These are the “pure” Hope and Despair contributions to the paragraph’s meaning.[71]

B.3 Phasor Geometry and Interpretation

The amplitudes

$$| c_H | = 0.78, | c_D | = 0.22$$

place the paragraph predominantly in the Hope direction, with a smaller Despair component. In a two-dimensional Hope–Despair plane, this corresponds to a point in a quadrant where Hope dominates but uncertainty remains.[72]

The phase difference $\phi_H - \phi_D$controls interference:

- If $\phi_H - \phi_D$is small, Hope and Despair contributions are nearly in phase, reinforcing a bittersweet tone.
- If $\phi_H - \phi_D \approx \pi$, Despair partially cancels Hope, yielding ambivalence.[73]

In the given paragraph, imagery such as "morning light" and "beginning" suggests that the Hope phase is aligned to yield constructive interference in the Hope direction.[74]

B.4 Optional Semantic-Phasor Rotation

If we apply a contextual operator encoding uncertainty,

$$U_{\text{uncertainty}} = e^{iH_{\text{uncertainty}}},$$

its effect on the Hope component might be

$$c'_H = e^{i\theta_H}\, c_H,$$

slightly rotating the Hope phase and thereby softening the confident tone into a more tentative one.[75]

This toy example demonstrates how the operator calculus provides a precise spectral method for analyzing emotional meaning in a low-dimensional subspace.[76]

Endnotes

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